A. Bay
Algebra 1

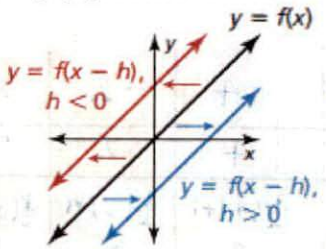
Name: Key
Date:

Transformations of Graphs of Linear Functions
3.7

A **family of functions** is a group of functions with similar characteristics. The most basic function in a family of functions is the **parent function**. For nonconstant linear functions, the parent function is $f(x) = x$ or $y = x$. The graphs of all other nonconstant linear functions are transformations of the graph of the parent function. A **transformation** changes the size, shape, position, or orientation of a graph.

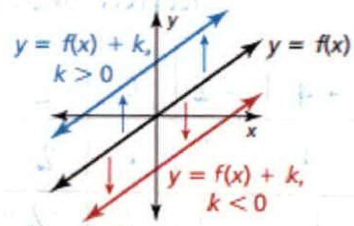
A **translation** is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations
The graph of $\boxed{y = f(x - h)}$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the inputs before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

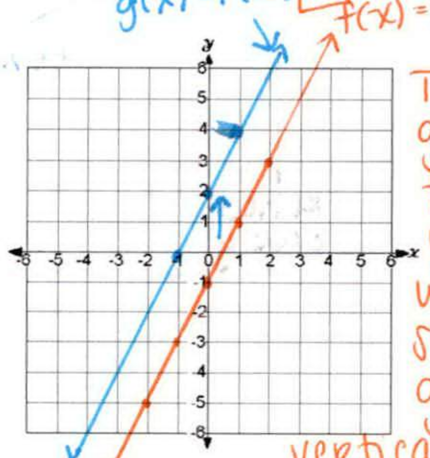
Vertical Translations
The graph of $\boxed{y = f(x) + k}$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



Adding k to the outputs shifts the graph down when $k < 0$ and up when $k > 0$.

Example 1: Horizontal and Vertical Translations
Let $\boxed{f(x) = 2x - 1}$. Graph (a) $\boxed{g(x) = f(x) + 3}$ and (b) $\boxed{t(x) = f(x + 3)}$. Describe the transformations from the graph of f to the graphs of g and t.

$g(x) = f(x)\boxed{+3}$ $\quad f(x) = 2x - 1$

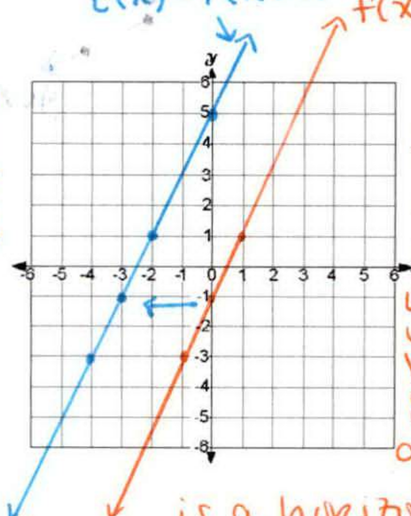


The function g is in the form $y = f(x) + k$ where $k = 3$ so the graph g is a vertical translation +3 or 3 units up from the graph f.

$g(x) = f(x) + 3$
$= 2x - 1 + 3$
$= 2x + 2$

* notice $t(x) = f(x+3)$
$= f(x - (-3))$
so $h = -3$

$t(x) = f(x+3)$ $\quad f(x) = 2x - 1$



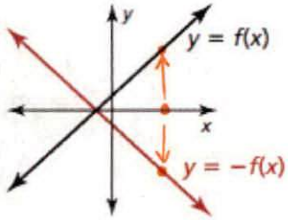
The function t is in the form $y = f(x - h)$ where $h = -3$ so the graph t is a horizontal translation −3 or 3 units to the left of graph f.

$t(x) = f(x+3)$
$= 2(x+3) - 1$
$= 2x + 6 - 1$
$= 2x + 5$

A **reflection** is a transformation that flips a graph over a line called the line of reflection.

Reflections in the x-axis

The graph of $\boxed{y = -f(x)}$ or is a reflection in the x-axis of the graph of $y = f(x)$.

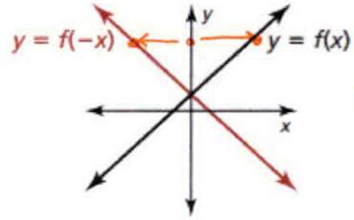


all points Reflect over the x-axis

Multiplying the outputs by –1 changes their signs.

Reflections in the y-axis

The graph of $\boxed{y = f(-x)}$ is a reflection in the y-axis of the graph of $y = f(x)$.



all points Reflect over the y-axis

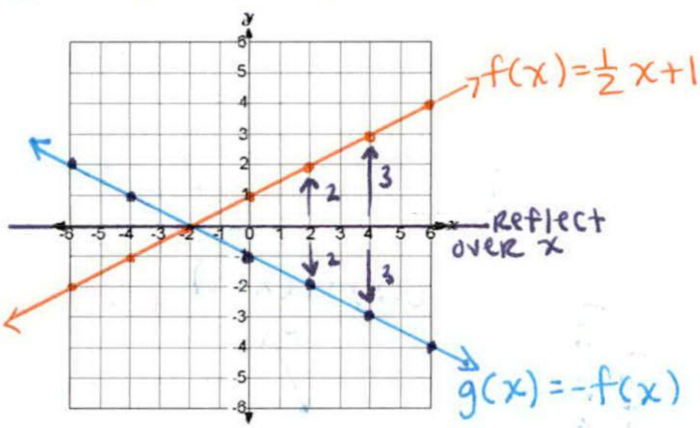
Multiplying the inputs by –1 changes their signs.

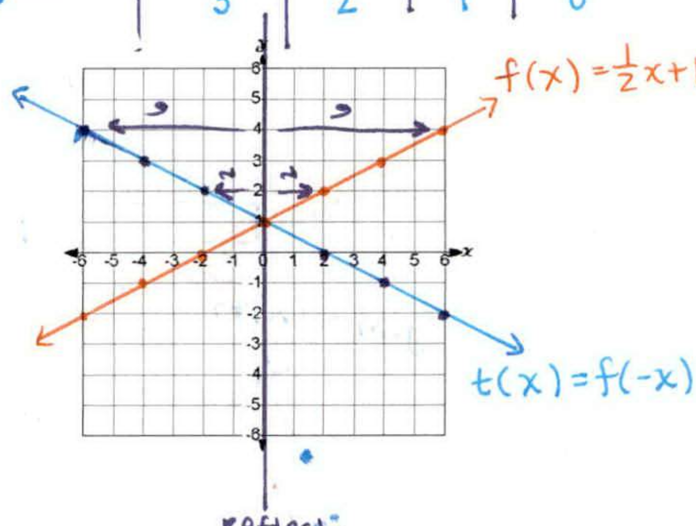
Example 2: Reflections in the x-axis and the y-axis

Let $\boxed{f(x) = \frac{1}{2}x + 1.}$ Graph (a) $\boxed{g(x) = -f(x)}$ and (b) $t(x) = f(-x)$. Describe the transformations from the graph of f to the graphs of g and t. Reflect over x-axis

x	-4	-2	0	2
$f(x)$	-1	0	1	2
$-f(x)/g(x)$	1	0	-1	-2

x	-4	-2	0	2
$-x$	4	2	0	-2
$t(x)/f(-x)$	$\frac{1}{2}(4)+1$ 3	$\frac{1}{2}(2)+1$ 2	$\frac{1}{2}(0)+1$ 1	$\frac{1}{2}(-2)+1$ 0





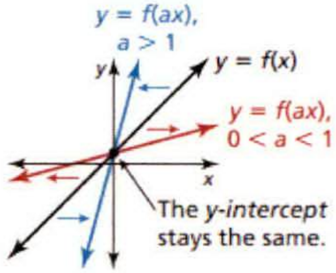
Stretches and Shrinks

You can transform a function by multiplying all the x-coordinates (inputs) by the same factor a. When $a > 1$, the transformation is a **horizontal shrink** because the graph shrinks toward the y-axis. When $0 < a < 1$, the transformation is a **horizontal stretch** because the graph stretches away from the y-axis. In each case, the y-intercept stays the same.

You can also transform a function by multiplying all the y-coordinates (outputs) by the same factor a. When $a > 1$, the transformation is a **vertical stretch** because the graph stretches away from the x-axis. When $0 < a < 1$, the transformation is a **vertical shrink** because the graph shrinks toward the x-axis. In each case, the x-intercept stays the same.

Horizontal Stretches and Shrinks

The graph of $\boxed{y = f(ax)}$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



$0 <$ factor < 1

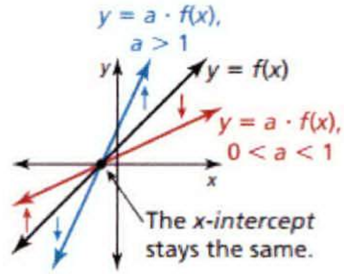
shrink

factor > 1

stretch

Vertical Stretches and Shrinks

The graph of $\boxed{y = a \cdot f(x)}$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



Example 3: Horizontal and Vertical Stretches

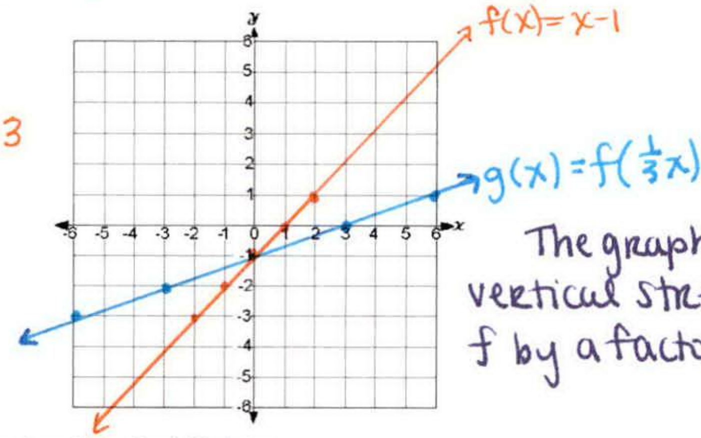
Let $\boxed{f(x) = x - 1}$. Graph (a) $\boxed{g(x) = f\left(\frac{1}{3}x\right)}$ and (b) $\boxed{h(x) = 3f(x)}$. Describe the transformations from the graph of f to the graphs of g and h.

$g(x) = f(\frac{1}{3}x)$
$= (\frac{1}{3}x) - 1$
$g(x) = \frac{1}{3}x - 1$

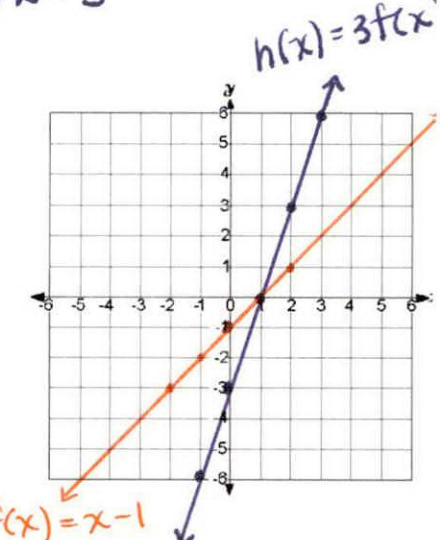
The graph of g is a horizontal stretch of f by a factor of $1 \div \frac{1}{3} = 3$.

$h(x) = 3f(x)$
$= 3(x-1)$
$h(x) = 3x - 3$

$a = \frac{1}{3}$
$\therefore \frac{1}{a} = \frac{1}{\frac{1}{3}} = 3$



The graph h is a vertical stretch of f by a factor of 3



Example 4: Horizontal and Vertical Shrinks

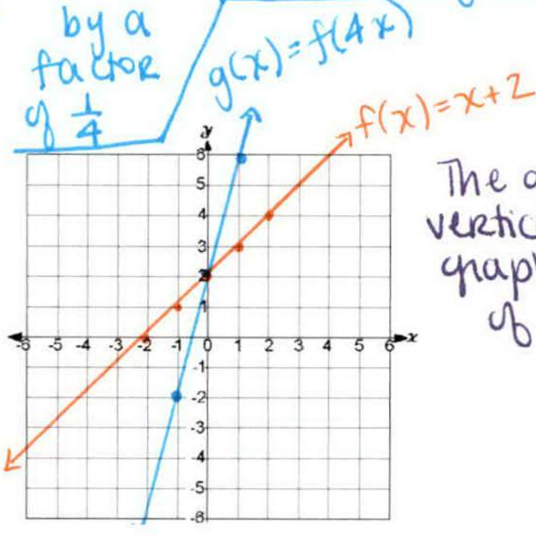
Let $\boxed{f(x) = x + 2}$. Graph (a) $\boxed{g(x) = f(4x)}$ and (b) $\boxed{h(x) = \frac{1}{4}f(x)}$. Describe the transformations from the graph of f to the graphs of g and h.

$g(x) = f(4x)$
$= (4x) + 2$
$g(x) = 4x + 2$

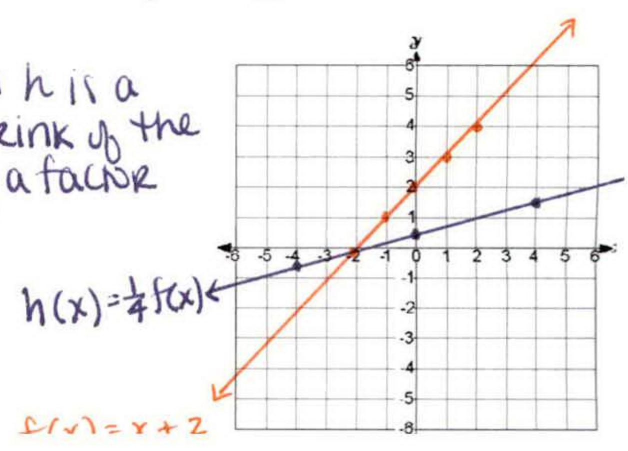
The graph of g is a horizontal shrink of f by a factor of $\frac{1}{4}$

$h(x) = \frac{1}{4}f(x)$
$= \frac{1}{4}(x+2)$
$h(x) = \frac{1}{4}x + \frac{1}{2}$

$a = 4$
$\therefore \frac{1}{a} = \frac{1}{4}$



The graph h is a vertical shrink of the graph by a factor of $\frac{1}{4}$



Transformations of Graphs

The graph of $y = a \cdot f(x-h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

Step 1: Translate the graph of $y = f(x)$ horizontally h units.

Step 2: Use a to stretch or shrink the resulting graph from Step 1.

Step 3: Reflect the resulting graph from Step 2 when $a < 0$.

Step 4: Translate the resulting graph from Step 3 vertically k units.

Parent function

Example 5: Combining Transformations

Graph $\boxed{f(x) = x}$ and $g(x) = -2x + 3$. Describe the transformations from the graph of f to the graphs of g and h.

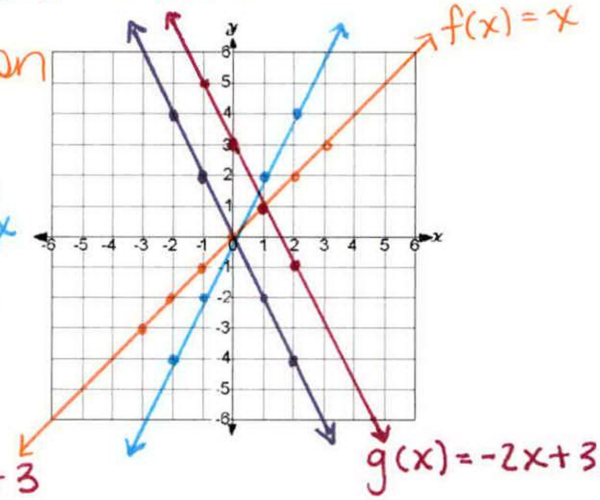
Note: $g(x) = -2x+3$ can be rewritten as $g(x) = -2f(x) + 3$

STEP 1: There is no horizontal translation from f to g

STEP 2: There is a vertical stretch of f by a factor of 2 to get $h(x) = 2x$

STEP 3: Reflect the graph of h over the x-axis to get $R(x) = -2x$

STEP 4: Translate the graph of R up by 3 units (vertically) to get $g(x) = -2x+3$

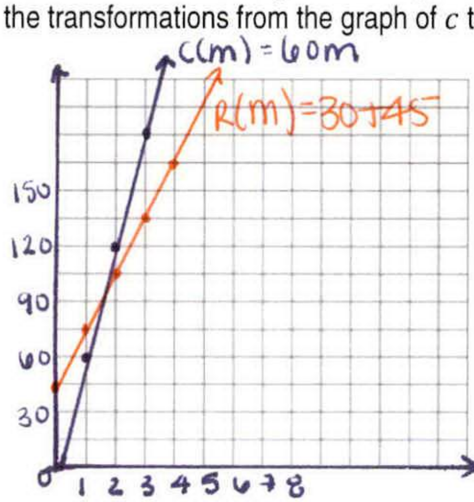


Example 6: Solving a Real-Life Problem

A cable company charges customers \$60 per month for its service, with no installation fee. The cost to a customer is represented by $c(m) = 60m$, where m is the number of months of service. To attract new customers, the cable company reduces the monthly fee to \$30 but adds an installation fee of \$45. The cost to a new customer is represented by $r(m) = 30m + 45$, where m is the number of months of service. Describe the transformations from the graph of c to the graph of r.

Note: $R(m) = 30m + 45$ can be rewritten as $R(m) = \frac{1}{2}c(m) + 45$

$\therefore$ the transformations are a vertical shrink by a factor of $\frac{1}{2}$ and a vertical translation up by 45 units



Homework: pg 145: 6-28 even, 36, 37, 41, 45