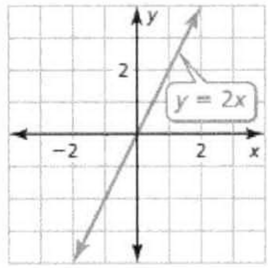
A. Bay
Algebra 1

Name: Key
Date:

Modeling Direct Variation
3.6

Two quantities x and y show **direct variation** when $y = ax$ and $a \neq 0$. The number a is called the **constant of variation**, and y is said to vary directly with x. The equation $y = 2x$ is an example of direct variation, and the constant of variation is 2.



$y = mx + 0$

$y = ax$

slope   y-int of (0,0)

Notice that a direct variation equation $y = ax$ is a linear equation in slope-intercept form, $y = mx + b$, with $m = a$ and $b = 0$. The graph of a direct variation equation is a line with a slope of a that passes through the origin.

Example 1: Determine whether x and y show direct variation. If so, identify the constant of variation

Do they fit the form $y = ax$?

a. $2x - 3y = 0$

$-2x \quad -2x$

$\frac{-3y}{-3} = \frac{-2x}{-3}$

$\boxed{y = \frac{2}{3}x}$

yes
$a = \frac{2}{3}$

b. $-x + y = 1$

$+x \quad +x$

$\boxed{y = x + 1}$

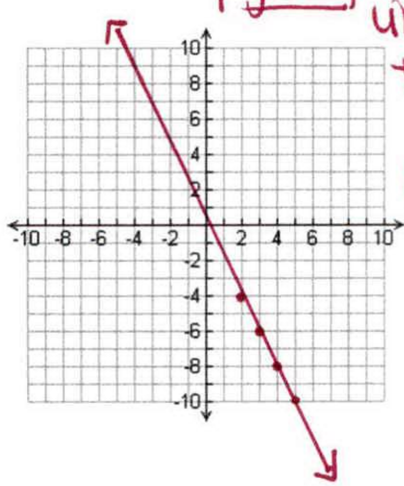
NO
line does not pass through the origin

Example 2: Determine whether x and y show direct variation. Explain.

a.

x	2	3	4	5
y	−4	−6	−8	−10

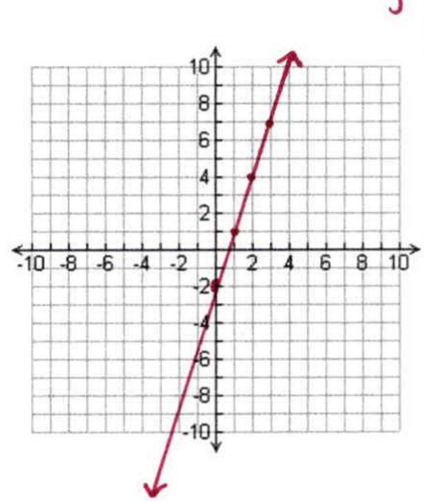
yes the line passes through the origin



b.

x	0	1	2	3
y	−2	1	4	7

NO the line has a y-intercept of (0, −2)



Example 3: Suppose y varies directly with x. meaning $y = ax$ OR $a = \frac{y}{x}$

* direct variation is proportional *

a. If $x = 27$ when $y = 6$, find x when $y = 2$.

notice $\frac{x}{y}$

$$\frac{27}{6} = \frac{x}{2}$$

$$27 \cdot 2 = 6x$$

$$\frac{54}{6} = \frac{6x}{6}$$

$$\boxed{x = 9}$$

$$\frac{1}{x} = \frac{6}{27} = \frac{2}{9} = a \qquad \boxed{y = \frac{2}{9}a}$$

b. If $y = -4$ when $x = 32$, find y when $x = 3$.

notice $\frac{y}{x}$

$$\frac{-4}{32} = \frac{y}{3}$$

$$-4 \cdot 3 = 32y$$

$$\frac{-12}{32} = \frac{32y}{32}$$

$$\boxed{-\frac{3}{8} = y}$$

$$\frac{y}{x} = \frac{-4}{32} = -\frac{1}{8} = a \qquad \boxed{y = -\frac{1}{8}a}$$

Example 4: The ordered pair $(-3, -6)$ is a solution of a direct variation equation. Write the equation and identify the constant of variation.

$$y = ax$$

$$\frac{-6}{-3} = \frac{a(-3)}{-3}$$

$$2 = a$$

$$\boxed{y = 2x}$$

Example 5: Assuming y varies directly with x, find the missing value. $(30, 8)$ and $(x, 4)$.

$$\frac{y}{x} = \frac{8}{30} = \frac{4}{15} = a$$

$$\boxed{y = \frac{4}{15}a}$$

$$\frac{8}{30} = \frac{4}{x}$$

$$30 \cdot 4 = 8x$$

$$\frac{120}{8} = \frac{8x}{8}$$

$$\boxed{x = 15}$$

Example 7: The table shows the costs C (in dollars) of downloading s songs from a music website.

a. Explain why C varies directly with s.

b. Write a direct variation equation that relates s and C.

notice $\frac{y}{x}$

x Number of songs, s	y Cost (dollars), C
3	2.97
5	4.95
7	6.93

ⓐ Find if all Ratios are equal

$$\frac{2.97}{3} = 0.99 \qquad \frac{4.95}{5} = 0.99 \qquad \frac{6.93}{7} = 0.99$$

ⓑ $y = 0.99x$ OR $\boxed{C = 0.99s}$

Example 8: The number s of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number w of gallons of water in the tank. A pet shop owner recommends that you add 100 tablespoons of salt to a 20-gallon tank. How many tablespoons of salt should you add to a 30-gallon tank?

$$s = aw$$

$$\frac{y}{x} = \frac{s}{w} = \frac{100}{20} = 5 = a$$

$$\boxed{s = 5w}$$

$$\frac{100}{20} = \frac{s}{30}$$

$$30 \cdot 100 = 20s$$

$$\frac{3000}{20} = \frac{20s}{20}$$

$$150 = s$$

$\boxed{\text{150 tablespoons}}$

Homework: pg 137: 3-25 odd, 28